Mathematics

Paper 4

Probability

6 Suleika has six cards numbered 1 to 6.

(a) She takes one card at random, records the number and replaces the card.

(i) Write down the probability that the number is 5 or 6.

..... [1]

(ii) Suleika does this 300 times.

Find how many times she expects the number 5 or 6.

..... [1]

(b) Suleika takes two cards at random, without replacement.

(i) Find the probability that the sum of the numbers on the two cards is 5.

..... [3]

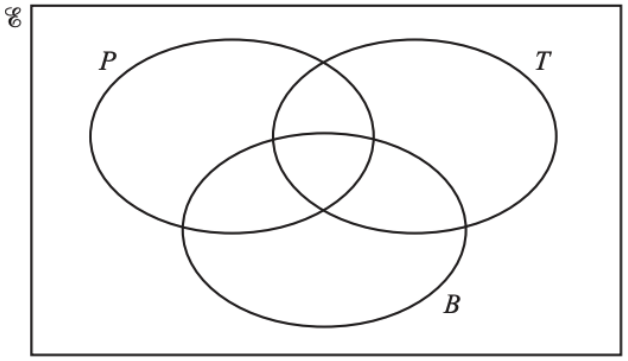
(ii) Find the probability that at least one of the numbers on the cards is a square number.

..... [3]

9 This year, 40 students have each travelled by one or more of plane (P), train (T) or boat (B).

7 have travelled only by plane.
11 have travelled only by train.
9 have travelled only by boat.

$n(P \cap T) = 8$
$n(B \cap T) = 3$
$n(B \cap P) = 6$



(d) Two students are chosen at random.

Calculate the probability that they both have travelled only by plane.

..... [2]

(e) Two students are chosen at random from those who have travelled by train.

Calculate the probability that they both have also travelled by plane.

..... [2]

2 The heights, h metres, of the 120 boys in an athletics club are recorded.
The table shows information about the heights of the boys.

Height (h metres)	$1.3 < h \leqslant 1.4$	$1.4 < h \leqslant 1.5$	$1.5 < h \leqslant 1.6$	$1.6 < h \leqslant 1.7$	$1.7 < h \leqslant 1.8$	$1.8 < h \leqslant 1.9$
Frequency	7	18	30	24	27	14

(b) (i) One boy is chosen at random from the club.

Find the probability that this boy has a height greater than 1.8 m.

..... [1]

(ii) Three boys are chosen at random from the club.

Calculate the probability that one of the boys has a height greater than 1.8 m and the other two boys each have a height of 1.4 m or less.

..... [4]

3 The speed, v km/h, of each of 200 cars passing a building is measured.
The table shows the results.

Speed (v km/h)	$0 < v \leqslant 20$	$20 < v \leqslant 40$	$40 < v \leqslant 45$	$45 < v \leqslant 50$	$50 < v \leqslant 60$	$60 < v \leqslant 80$
Frequency	16	34	62	58	26	4

(c) Two of the 200 cars are chosen at random.

Find the probability that they both have a speed greater than 50 km/h.

..... [2]

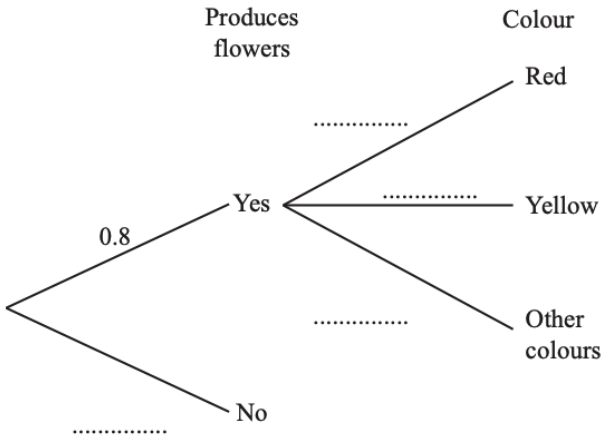
7 Tanya plants some seeds.
The probability that a seed will produce flowers is 0.8 .
When a seed produces flowers, the probability that the flowers are red is 0.6 and the probability that the flowers are yellow is 0.3 .

(a) Tanya has a seed that produces flowers.

Find the probability that the flowers are not red and not yellow.

..... [1]

(b) **(i)** Complete the tree diagram.



[2]

(ii) Find the probability that a seed chosen at random produces red flowers.

..... [2]

(iii) Tanya chooses a seed at random.

Find the probability that this seed does not produce red flowers and does not produce yellow flowers.

..... [3]

(c) Two of the seeds are chosen at random.

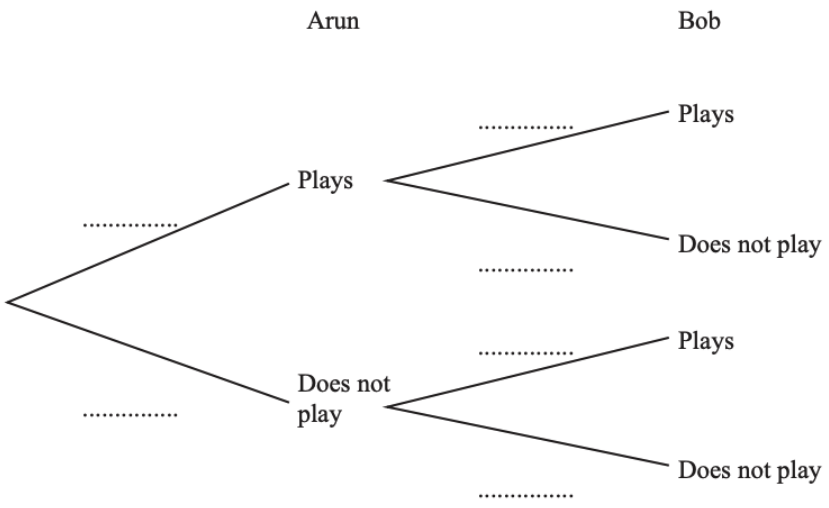
Find the probability that one produces flowers and one does not produce flowers.

..... [3]

7 On any Saturday, the probability that Arun plays football is $\frac{3}{4}$.

On any Saturday, the probability that Bob plays football is $\frac{2}{5}$.

(a) (i) Complete the tree diagram.



[2]

(ii) Calculate the probability that, one Saturday, Arun and Bob both play football.

..... [2]

(iii) Calculate the probability that, one Saturday, either Arun plays football or Bob plays football, but not both.

..... [3]

(b) Calculate the probability that Bob plays football for 2 of the next 3 Saturdays.

..... [3]

(c) When Arun plays football, the probability that he scores the winning goal is $\frac{1}{7}$.

Calculate the probability that Arun scores the winning goal one Saturday.

..... [2]

9.

(c) Another bag contains 15 red beads and 10 yellow beads.
Ariana picks a bead at random, records its colour and replaces it in the bag.
She then picks another bead at random.

(i) Find the probability that she picks two red beads.

..... [2]

(ii) Find the probability that she does not pick two red beads.

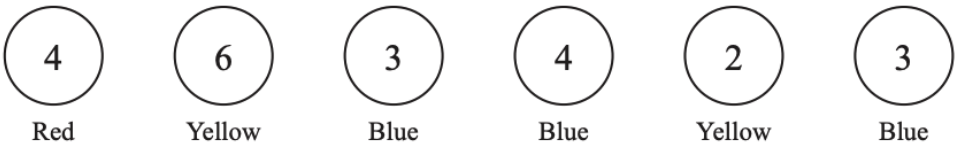
..... [1]

(d) A box contains 15 red pencils, 8 yellow pencils and 2 green pencils.
Two pencils are picked at random without replacement.

Find the probability that at least one pencil is red.

..... [3]

6



The diagram shows six discs.
Each disc has a colour and a number.

(a) One disc is picked at random.

Write down the probability that

(i) the disc has the number 4,

..... [1]

(ii) the disc is red and has the number 3,

..... [1]

(iii) the disc is blue and has the number 4.

..... [1]

(b) Two of the six discs are picked at random **without** replacement.

Find the probability that

(i) both discs have the number 3,

..... [2]

(ii) both discs have the same colour.

..... [3]

(c) Two of the six discs are picked at random **with** replacement.

Find the probability that both discs have the same colour.

..... [3]

4

P	O	S	S	I	B	I	L	I	T	Y

Morgan picks two of these letters, at random, **without** replacement.

(a) Find the probability that he picks

(i) the letter Y first,

..... [1]

(ii) the letter B then the letter Y,

..... [2]

(iii) two letters that are the same.

..... [3]

(b) Morgan now picks a third letter at random.

Find the probability that

(i) all three letters are the same,

..... [2]

(ii) exactly two of the three letters are the same,

..... [5]

(iii) all three letters are different.

..... [2]

3 Sushila, Ravi and Talika each have a bag of balls.
Each of the bags contains 10 red balls and 8 blue balls.

(a) Sushila takes one ball at random from her bag.

Find the probability that she takes a red ball.

..... [1]

(b) Ravi takes two balls at random from his bag, without replacement.

Find the probability that one ball is red and one ball is blue.

..... [3]

(c) Talika takes three balls at random from her bag, without replacement.

Calculate the probability that the three balls are the same colour.

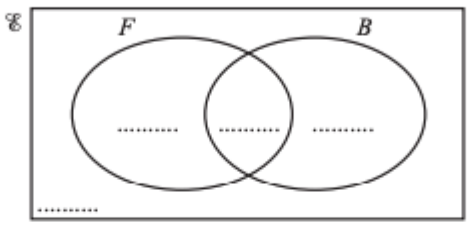
..... [4]

6 $\mathscr{E}$ = {students in a school}
F = {students who play football}
B = {students who play baseball}

There are 240 students in the school.

- 120 students play football
- 40 students play baseball
- 90 students play football but not baseball.

(a) Complete the Venn diagram to show this information.



[2]

(b) Find $n(F' \cap B')$.

..... [1]

(c) A student in the school is chosen at random.

Find the probability that this student plays baseball but not football.

..... [1]

(d) Two students who play baseball are chosen at random.

Find the probability that they both also play football.

..... [3]

3 The probability that Andrei cycles to school is r.

(a) Write down, in terms of r, the probability that Andrei **does not** cycle to school.

..... [1]

(b) The probability that Benoit **does not** cycle to school is $1.3 - r$.
The probability that both Andrei and Benoit **do not** cycle to school is 0.4 .

(i) Complete the equation in terms of r.

(.....) × (.....) = 0.4 [1]

(ii) Show that this equation simplifies to $10r^2 - 23r + 9 = 0$.

[3]

(iii) Solve by factorisation $10r^2 - 23r + 9 = 0$.

$r =$ ..... or $r =$ ..... [3]

(iv) Find the probability that Benoit **does not** cycle to school.

..... [1]

8 **(a)** Angelo has a bag containing 3 white counters and x black counters.
He takes two counters at random from the bag, without replacement.

(i) Complete the following statement.

The probability that Angelo takes two black counters is

$$\frac{x}{x+3} \times \frac{\text{.....}}{\text{.....}}.$$

[2]

(ii) The probability that Angelo takes two black counters is $\frac{7}{15}$.

(a) Show that $4x^2 - 25x - 21 = 0$.

[4]

(b) Solve by factorisation.

$$4x^2 - 25x - 21 = 0$$

$x =$ ..... or $x =$ ..... [3]

(c) Write down the number of black counters in the bag.

..... [1]

(b) Esme has a bag with 5 green counters and 4 red counters.
She takes three counters at random from the bag without replacement.

Work out the probability that the three counters are all the same colour.

..... [4]

8

The diagram shows 5 cards.

(a) Donald chooses a card at random.

(i) Write down the probability that the number of dots on this card is an even number.

..... [1]

(ii) Write down the probability that the number of dots on this card is a prime number.

..... [1]

(b) Donald chooses two of the five cards at random, without replacement.
He works out the total number of dots on these two cards.

(i) Find the probability that the total number of dots is 5.

..... [3]

(ii) Find the probability that the total number of dots is an odd number.

..... [3]

8 **(a)** A bag contains 4 red marbles and 2 yellow marbles.
Behnaz picks two marbles at random without replacement.

Find the probability that

(i) the marbles are both red,

..... [2]

(ii) the marbles are not both red.

..... [1]

(b) Another bag contains 5 blue marbles and 2 green marbles.
Bryn picks one marble at random without replacement.
If this marble is not green, he picks another marble at random without replacement.
He continues until he picks a green marble.

Find the probability that he picks a green marble on his first, second or third attempt.

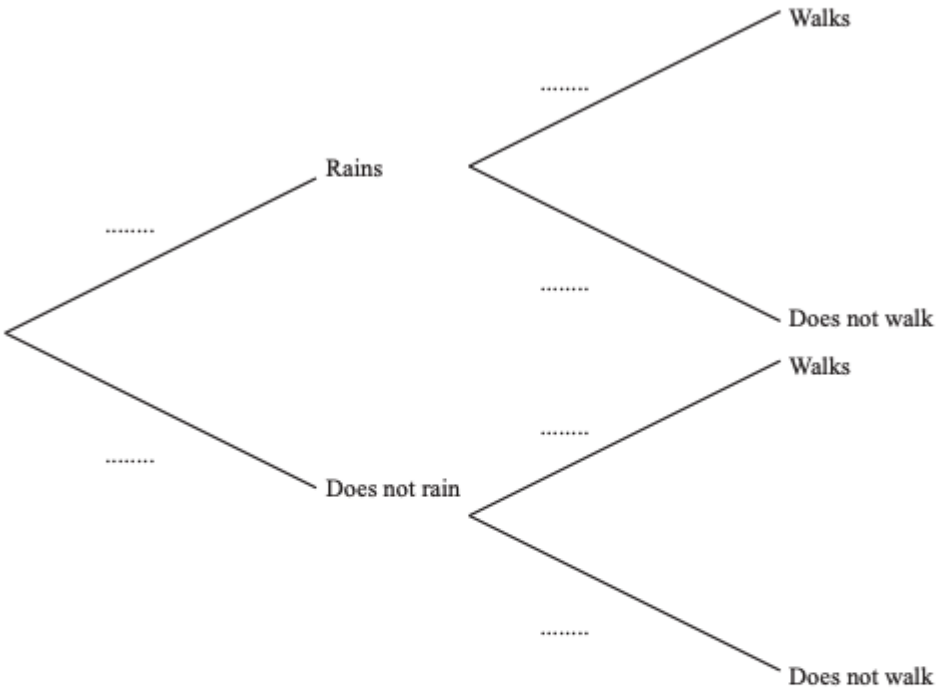
..... [4]

9 The probability that it will rain tomorrow is $\frac{5}{8}$.

If it rains, the probability that Rafael walks to school is $\frac{1}{6}$.

If it does not rain, the probability that Rafael walks to school is $\frac{7}{10}$.

(a) Complete the tree diagram.



[3]

(b) Calculate the probability that it will rain tomorrow and Rafael walks to school.

..... [2]

(c) Calculate the probability that Rafael does not walk to school.

..... [3]

4 **(a)** The diagram shows two sets of cards.

Set *A*	1	1	2	2	2
Set *B*	0	1	1	1	2

(i) Jojo chooses two cards at random from Set *A* without replacement.

Find the probability that the two cards have the same number.

..... [3]

(ii) Jojo replaces the two cards.
Kylie then chooses one card at random from Set *A* and one card at random from Set *B*.

Find the probability that the two cards have the same number.

..... [3]

(iii) Who is the most likely to choose two cards that have the same number?
Show all your working.

..... [1]

(b)

Set C

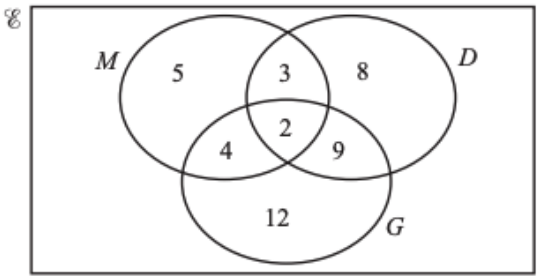
4	4	5	5	5

Lena chooses three cards at random from Set C without replacement.

Find the probability that the third card chosen is numbered 4.

..... [3]

6 (a)



The Venn diagram above shows information about the number of students who study Music (M), Drama (D) and Geography (G).

(i) How many students study Music?

..... [1]

(ii) How many students study exactly two subjects?

..... [1]

(iii) Two students are chosen at random from those who study Drama.

Calculate the probability that they both also study Music.

..... [3]

12 A box contains 20 packets of potato chips.

6 packets contain barbecue flavoured chips.
10 packets contain salt flavoured chips.
4 packets contain chicken flavoured chips.

(a) Maria takes two packets at random **without replacement**.

(i) Show that the probability that she takes two packets of salt flavoured chips is $\frac{9}{38}$.

[2]

(ii) Find the probability that she takes two packets of different flavoured chips.

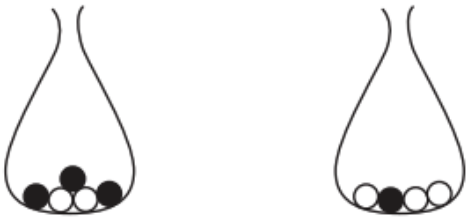
..... [4]

(b) Maria takes three packets at random, **without replacement**, from the 20 packets.

Find the probability that she takes **at least two** packets of chicken flavoured chips.

..... [3]

7



Bag A Bag B

Bag A contains 3 black balls and 2 white balls.
Bag B contains 1 black ball and 3 white balls.

(a) A ball is taken at random from each bag.

(i) Show that a black ball is more likely to be taken from bag A than from bag B.

[1]

(ii) Find the probability that the two balls have different colours.

..... [3]

(b) The balls are returned to their original bags.
Three balls are taken at random from bag A, without replacement.

Find the probability that

(i) they are all black,

..... [2]

(ii) they are all white.

..... [1]

(c) The balls are returned to their original bags.

A ball is taken at random from bag A and its colour is recorded.
This ball is then placed in bag B.
A ball is then taken at random from bag B.

Find the probability that the ball taken from bag B has a different colour to the ball taken from bag A.

..... [3]

4 Ravi spins a biased 5-sided spinner, numbered 1 to 5.
The probability of each number is shown in the table.

Number	1	2	3	4	5
Probability	$\frac{1}{6}$	$\frac{1}{4}$	$\frac{1}{3}$	x	x

(a) Find the value of x.

$x =$ ..... [3]

(b) Ravi spins the spinner once.

Find the probability that the number is 2 or 3.

..... [2]

(c) Ravi spins the spinner twice.

Find the probability that

(i) the number is 2 both times,

..... [2]

(ii) the sum of the numbers is 3.

..... [3]

(d) Ravi spins the spinner 72 times.

Calculate how many times he expects the number 1.

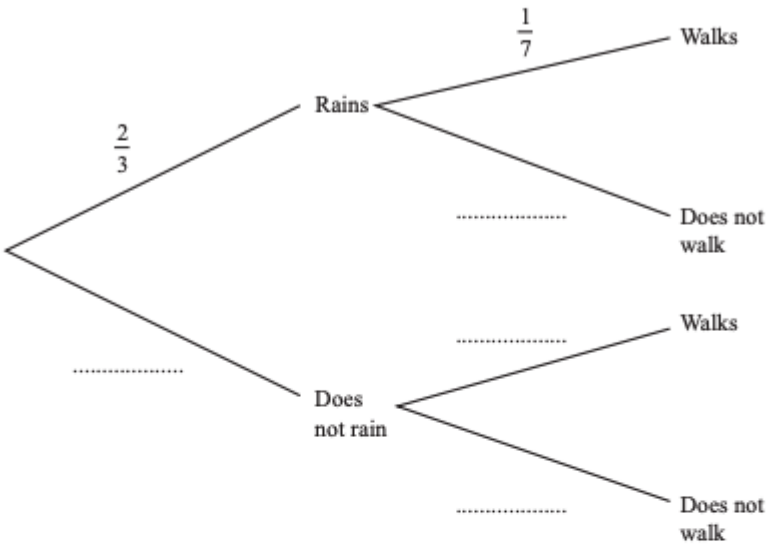
..... [1]

6 Each morning the probability that it rains is $\frac{2}{3}$.

If it rains, the probability that Asha walks to school is $\frac{1}{7}$.

If it does not rain, the probability that Asha walks to school is $\frac{4}{7}$.

(a) Complete the tree diagram.



[2]

(b) Find the probability that it rains and Asha walks to school.

..... [2]

(c) **(i)** Find the probability that Asha does not walk to school.

..... [3]

(ii) Find the expected number of days Asha does not walk to school in a term of 70 days.

..... [2]

(d) Find the probability that it rains on exactly one morning in a school week of 5 days.

..... [2]

9 **(a)** A bag contains red beads and green beads.
There are 80 beads altogether.
The probability that a bead chosen at random is green is 0.35 .

(i) Find the number of red beads in the bag.

..... [2]

(ii) Marcos chooses a bead at random and replaces it in the bag.
He does this 240 times.

Find the number of times he would expect to choose a green bead.

..... [1]

(b) A different bag contains 2 blue marbles, 3 yellow marbles and 4 white marbles.
Huma chooses a marble at random, notes the colour, then replaces it in the bag.
She does this three times.

Find the probability that

(i) all three marbles are yellow,

..... [2]

(ii) all three marbles are different colours.

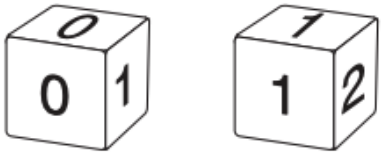
..... [3]

(c) Another bag contains 2 green counters and 3 pink counters.
Teresa chooses three counters at random **without** replacement.

Find the probability that she chooses more pink counters than green counters.

..... [4]

7



A *B*

The diagram shows two fair dice.
The numbers on dice *A* are 0, 0, 1, 1, 1, 3.
The numbers on dice *B* are 1, 1, 2, 2, 2, 3.
When a dice is rolled, the score is the number on the top face.

(a) Dice *A* is rolled once.

Find the probability that the score is not 3.

..... [1]

(b) Dice *A* is rolled twice.

Find the probability that the score is 0 both times.

..... [2]

(c) Dice *A* is rolled 60 times.

Calculate an estimate of the number of times the score is 0.

..... [1]

(d) Dice A and dice B are each rolled once.
The product of the scores is recorded.

(i) Complete the possibility diagram.

Dice B \ Dice A	0	0	1	1	1	3
3	0	0				
2	0	0				
2	0	0				
2	0	0				
1	0	0				
1	0	0	1	1	1	3

[2]

(ii) Find the probability that the product of the scores is

(a) 2,

..... [1]

(b) greater than 3.

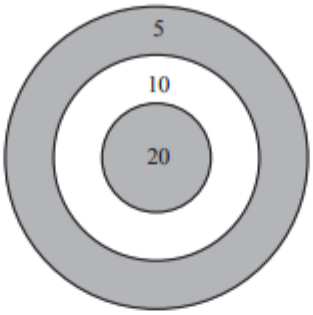
..... [1]

(e) Eva keeps rolling dice B until 1 is scored.

Find the probability that this happens on the 5th roll.

..... [2]

5 Kiah plays a game.
The game involves throwing a coin onto a circular board.
Points are scored for where the coin lands on the board.



If the coin lands on part of a line or misses the board then 0 points are scored.
The table shows the probabilities of Kiah scoring points on the board with one throw.

Points scored	20	10	5	0
Probability	x	0.2	0.3	0.45

(a) Find the value of x.

$x =$ ..... [2]

(b) Kiah throws a coin fifty times.

Work out the expected number of times she scores 5 points.

..... [1]

(c) Kiah throws a coin two times.

Calculate the probability that

(i) she scores either 5 or 0 with her first throw,

..... [2]

(ii) she scores 0 with her first throw and 5 with her second throw,

..... [2]

(iii) she scores a total of 15 points with her two throws.

..... [3]

(d) Kiah throws a coin three times.

Calculate the probability that she scores a total of 10 points with her three throws.

..... [5]

4 Coins are put into a machine to pay for parking cars.
The probability that the machine rejects a coin is 0.05 .

(a) Adhira puts 2 coins into the machine.

(i) Calculate the probability that the machine rejects **both** coins.

.....[2]

(ii) Calculate the probability that the machine accepts at **least one** coin.

.....[1]

(b) Raj puts 4 coins into the machine.

Calculate the probability that the machine rejects **exactly one** coin.

.....[3]

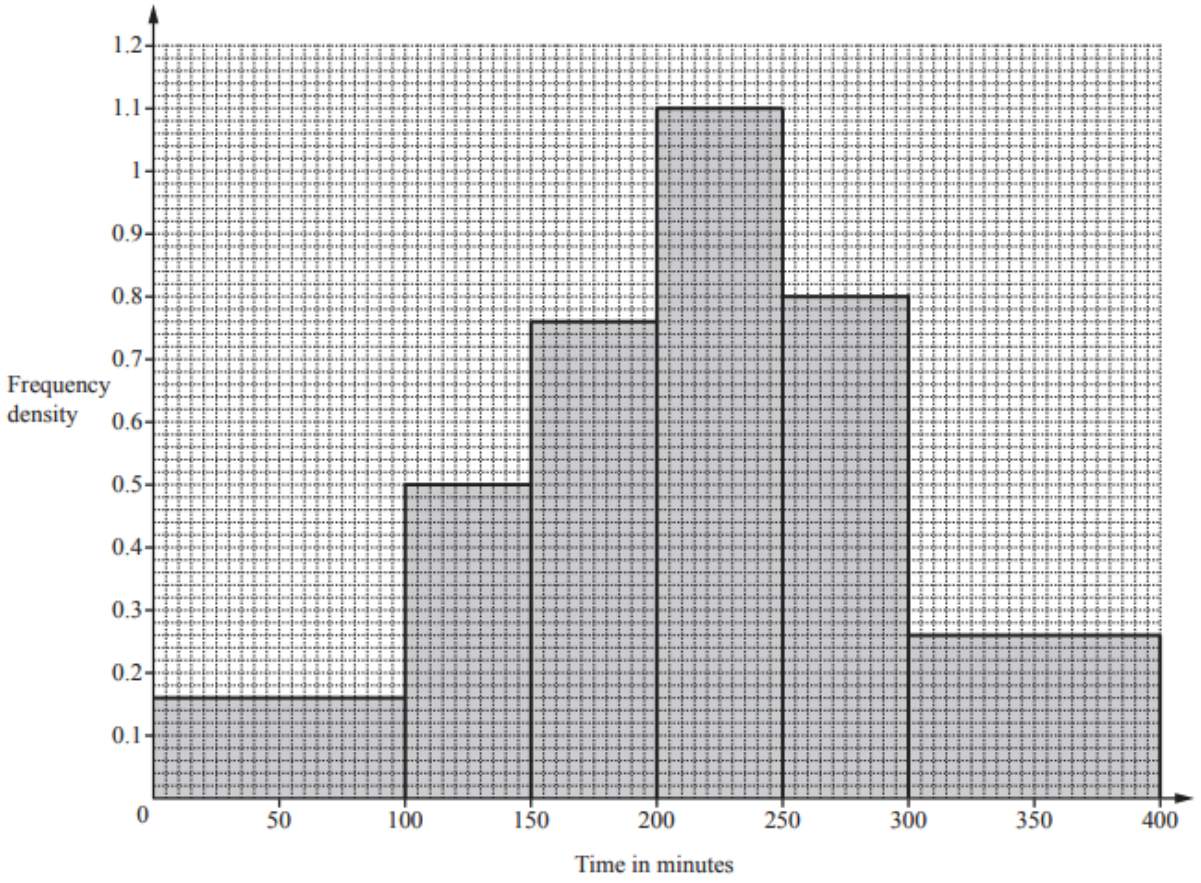
(c) The table shows the amount of money, $\$a$, received for parking each day for 200 days.

Amount ($\$a$)	$200 < a \leqslant 250$	$250 < a \leqslant 300$	$300 < a \leqslant 350$	$350 < a \leqslant 400$	$400 < a \leqslant 450$	$450 < a \leqslant 500$
Frequency	13	19	27	56	62	23

Calculate an estimate of the mean amount of money received each day.

$.....[4]

(d) The histogram shows the length of time that 200 cars were parked.



(i) Calculate the number of cars that were parked for 100 minutes or less.

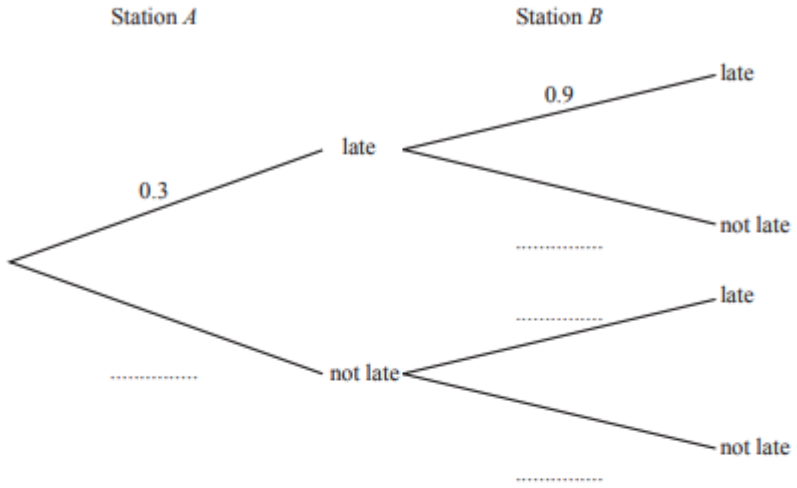
..... [1]

(ii) Calculate the percentage of cars that were parked for more than 250 minutes.

.....% [2]

7 A train stops at station A and then at station B.
If the train is late at station A, the probability that it is late at station B is 0.9 .
If the train is not late at station A, the probability that it is late at station B is 0.2 .
The probability that the train is late at station A is 0.3 .

(a) Complete the tree diagram.



[2]

(b) **(i)** Find the probability that the train is late at one or both of the stations.

.....[3]

(ii) This train makes 250 journeys.

Find the number of journeys that the train is expected to be late at one or both of the stations.

.....[1]

(c) The train continues to station C.
The probability that it is late at all 3 stations is 0.27 .

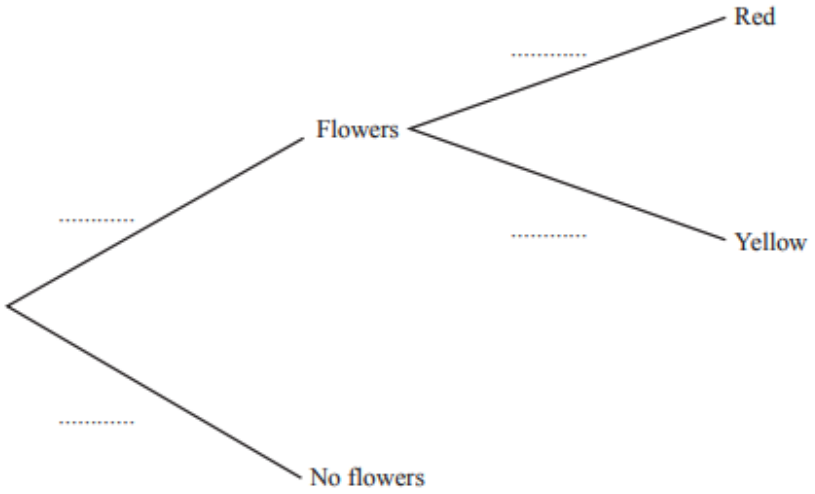
Describe briefly what this probability shows.

.....

.....[1]

5 The probability that a plant will produce flowers is $\frac{7}{8}$.
The flowers are either red or yellow.
If the plant produces flowers, the probability that the flowers are red is $\frac{3}{4}$.

(a) (i) Complete the tree diagram by writing a probability beside each branch.



[2]

(ii) Calculate the probability that a plant, chosen at random, will produce red flowers.

..... [2]

(iii) Two plants are chosen at random.

Calculate the probability that both will produce red flowers.

..... [2]

(b) Alphonse buys 200 of these plants.

Calculate the number of plants that are expected to produce flowers.

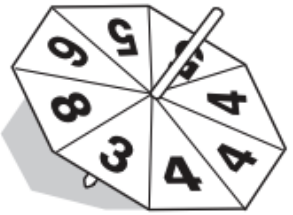
..... [2]

(c) Gabriel has 1575 plants with red flowers.

Estimate the total number of plants that Gabriel has.

..... [2]

5 Sandra has a fair eight-sided spinner.
The numbers on the spinner are 3, 4, 4, 4, 5, 5, 6 and 8.
Sandra spins the spinner twice and records each number it lands on.



Find the probability that

(a) both numbers are 8,

..... [2]

(b) the two numbers are not both 8,

..... [1]

(c) one number is odd and one number is even,

..... [2]

(d) the total of the two numbers is at least 13,

..... [3]

(e) the second number is bigger than the first number.

..... [3]

6(a)(i)	$\frac{1}{3}$ oe	**1**	
6(a)(ii)	100	**1**	**FT** *their* **(a)(i)** × 300 to at least 3 sf or rounded to the nearest integer
6(b)(i)	$\frac{2}{15}$ oe	**3**	**M2** for $4\times\frac{1}{6}\times\frac{1}{5}$ oe or **M1** for $k\left(\frac{1}{6}\times\frac{1}{5}\right)$ oe or list or indication of 4 correct pairs
6(b)(ii)	$\frac{3}{5}$ oe	**3**	**M2** for $1-\frac{4}{6}\times\frac{3}{5}$ or $2\left(\frac{2}{6}\times\frac{4}{5}\right)+\frac{2}{6}\times\frac{1}{5}$ oe or $\frac{2}{6}+\left(\frac{4}{6}\times\frac{2}{5}\right)$ oe or **M1** for $\frac{4}{6}\times\frac{3}{5}$ oe seen or $\frac{2}{6}\times\frac{4}{5}[\times 2]$ oe seen or $\frac{2}{6}\times\frac{1}{5}$ oe seen or correct identification of 18 pairs or space diagram oe

9(d)	$\frac{7}{260}$ oe	**2**	**M1** for $\frac{7}{40}\times\frac{6}{39}$ oe
9(e)	$\frac{14}{95}$ oe	**2**	**FT** *their* Venn diagram **M1** for $\frac{8}{20}\times\frac{7}{19}$

2(b)(i)	$\frac{14}{120}$ oe	**1**	
2(b)(ii)	$\frac{21}{20060}$ oe	**4**	**M3** for $3\left(\frac{14}{120}\times\frac{7}{119}\times\frac{6}{118}\right)$ or **M2** for $\frac{14}{120}\times\frac{7}{119}\times\frac{6}{118}$ isw or **M1** for $\frac{14}{120},\frac{7}{119},\frac{6}{118}$ After 0 scored, **SC1** for answer $\frac{343}{864000}$ or $\frac{343}{288000}$ oe

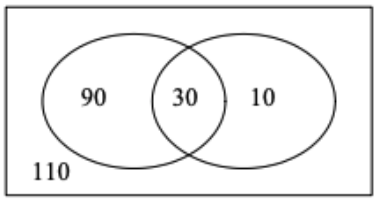
3(c)	$\frac{87}{3980}$ oe	**2**	**M1** for $\frac{30}{200}\times\frac{29}{199}$ oe

7(a)	0.1	1	
7(b)(i)	0.2 oe 0.6, 0.3, 0.1 oe	**2**	**B1** for 0.2 **B1** for 0.6, 0.3, 0.1
7(b)(ii)	0.48 oe	**2**	**FT** *their* 0.6 from tree diagram **M1** for $0.8 \times$ *their* 0.6
7(b)(iii)	0.28 oe	**3**	**M2** for $0.2 + 0.8 \times 0.1$ oe or **M1** for 0.2 or 0.8×0.1 or $0.8 \times (0.6 + 0.3)$
7(c)	0.32 oe	**3**	**M2** for $0.8 \times 0.2 + 0.2 \times 0.8$ oe **M1** for one of these products
7(a)(i)	$\frac{3}{4}, \frac{1}{4}$ $\frac{2}{5}, \frac{3}{5}$ $\frac{2}{5}, \frac{3}{5}$	**2**	**B1** for one correct pair
7(a)(ii)	$\frac{3}{10}$ oe	**2**	**FT** *their* tree diagram **M1** for $\frac{3}{4} \times \frac{2}{5}$
7(a)(iii)	$\frac{11}{20}$ oe	**3**	**M2** for $\frac{3}{4} \times \frac{3}{5} + \frac{1}{4} \times \frac{2}{5}$ or **M1** for $\frac{3}{4} \times \frac{3}{5}$ or $\frac{1}{4} \times \frac{2}{5}$
7(b)	$\frac{36}{125}$ oe	**3**	**M2** for $\left(\frac{2}{5}\right)^2 \times \frac{3}{5} \times 3$ oe or **M1** for $\left(\frac{2}{5}\right)^2 \times \frac{3}{5}$
7(c)	$\frac{3}{28}$ oe	**2**	**M1** for $\frac{3}{4} \times \frac{1}{7}$
9(c)(i)	$\frac{9}{25}$ oe	**2**	**M1** for $\frac{15}{25} \times \frac{15}{25}$ oe
9(c)(ii)	$\frac{16}{25}$ oe	**1**	**FT** $1 -$ *their* **(c)(i)**
9(d)	$\frac{17}{20}$ oe	**3**	**M2** for $1 - \frac{10}{25} \times \frac{9}{24}$ oe or for $\frac{15}{25} \times \frac{14}{24} + \frac{15}{25} \times \frac{8}{24} + \frac{15}{25} \times \frac{2}{24} + \frac{8}{25} \times \frac{15}{24} + \frac{2}{25} \times \frac{15}{24}$ oe or **M1** for one correct relevant product

6(a)(i)	$\frac{1}{3}$ oe	**1**	
6(a)(ii)	0	**1**	
6(a)(iii)	$\frac{1}{6}$ oe	**1**	
6(b)(i)	$\frac{1}{15}$ oe	**2**	**M1** for $\frac{2}{6}\times\frac{1}{5}$ or equivalent method
6(b)(ii)	$\frac{4}{15}$ oe	**3**	**M2** for $\frac{2}{6}\times\frac{1}{5}+\frac{3}{6}\times\frac{2}{5}$ or equivalent method or **M1** for $\frac{2}{6}\times\frac{1}{5}$ oe seen or $\frac{3}{6}\times\frac{2}{5}$ oe seen
6(c)	$\frac{7}{18}$ oe	**3**	**M2** for $\left(\frac{1}{6}\right)^2+\left(\frac{2}{6}\right)^2+\left(\frac{3}{6}\right)^2$ oe or **M1** for one correct product seen or sample space with 14 correct pairs identified

4(a)(i)	$\frac{1}{11}$ oe	**1**	
4(a)(ii)	$\frac{1}{110}$ oe	**2**	**M1** for $\frac{1}{11}\times\frac{1}{10}$ oe
4(a)(iii)	$\frac{4}{55}$ oe	**3**	**M2** for $\left(\frac{2}{11}\times\frac{1}{10}\right)+\left(\frac{3}{11}\times\frac{2}{10}\right)$ oe **or M1** for $\left(\frac{2}{11}\times\frac{1}{10}\right)$ or $\left(\frac{3}{11}\times\frac{2}{10}\right)$ seen oe
4(b)(i)	$\frac{1}{165}$ oe	**2**	**M1** for $\frac{3}{11}\times\frac{2}{10}\times\frac{1}{9}$ oe
4(b)(ii)	$\frac{1}{5}$ oe	**5**	**M4** for $3\left(\frac{2}{11}\times\frac{1}{10}\times\left[\frac{9}{9}\right]\right)+3\left(\frac{3}{11}\times\frac{2}{10}\times\frac{8}{9}\right)$ oe or **M3** for $3\left(\frac{3}{11}\times\frac{2}{10}\times\frac{8}{9}\right)$ or **M2** for $3\left(\frac{2}{11}\times\frac{1}{10}\times\left[\frac{9}{9}\right]\right)$ or $\frac{3}{11}\times\frac{2}{10}\times\frac{8}{9}$ oe or **M1** for $\frac{2}{11}\times\frac{1}{10}\times\left[\frac{k}{9}\right]$ where k is 3, 6 or 9

3(a)	$\frac{5}{9}$ oe	**1**	
3(b)	$\frac{80}{153}$ oe	**3**	**M2** for $2\times\frac{10}{18}\times\frac{8}{17}$ oe or **M1** for $\frac{10}{18}\times\frac{8}{17}$ oe If 0 scored, **SC1** for $\frac{160}{324}$ oe
3(c)	$\frac{11}{51}$ oe	**4**	**M3** for $\frac{10}{18}\times\frac{9}{17}\times\frac{8}{16}+\frac{8}{18}\times\frac{7}{17}\times\frac{6}{16}$ oe or **M2** for $\frac{10}{18}\times\frac{9}{17}\times\frac{8}{16}$ oe or $\frac{8}{18}\times\frac{7}{17}\times\frac{6}{16}$ oe or **M1** for $\frac{10}{18},\frac{9}{17},\frac{8}{16}$ or $\frac{8}{18},\frac{7}{17},\frac{6}{16}$ If 0 scored, **SC1** for $\frac{1512}{5832}$ oe

6(a)		**2**	**B1** for any one correct
6(b)	110	**1**	**FT** *their* 110 in Venn diagram
6(c)	$\frac{10}{240}$ oe	**1**	**FT** $\frac{their10}{240}$

6(d)	$\frac{870}{1560}$ oe	**3**	**M2** for $\frac{their30}{40}\times\frac{their30-1}{39}$ or **M1** for $\frac{p}{q}\times\frac{p-1}{q-1}$ $p<q$ or for $\frac{their30}{40}$ soi

3(a)	$1-r$	**1**	
3(b)(i)	$(1-r)(1.3-r)$ [= 0.4]	**1**	**FT** *their*(**a**) dep on **(a)** being an expression in r
3(b)(ii)	$1.3-1.3r-r+r^2$ or better nfww	**M1**	**FT** *their* **(b)(i)**
	$0.9-2.3r+r^2[=0]$ OR $13-13r-10r+10r^2=4$ oe	**M1**	**Strict FT** *their* expansion to a quadratic then equating to 0.4 and then collecting to 3 terms on 'one side' OR **Strict FT** *their* expansion to a quadratic = 0.4 all multiplied by 10
	$10r^2-23r+9=0$	**A1**	no errors or omissions seen
3(b)(iii)	$(5r-9)(2r-1)$ [= 0]	**B2**	or **B2** for e.g. $5r(2r-1)-9(2r-1)$ **and then** $5r-9=0$ **and** $2r-1=0$ or **B1** for $5r(2r-1)-9(2r-1)$ [= 0] or $2r(5r-9)-1(5r-9)$ [= 0] or $(5r+a)(2r+b)$ [= 0] where a, b are integers and $ab=+9$ or $2a+5b=-23$ If 0 scored, **SC1** for $5r-9$ **and** $2r-1$ seen but not in factorised form
	[$r=$] $\frac{9}{5}$ oe [$r=$] $\frac{1}{2}$ oe	**B1**	
3(b)(iv)	0.8 or $\frac{4}{5}$ oe	**1**	
8(a)(i)	$\frac{x-1}{x+2}$	**2**	**B1** for either numerator or denominator correct
8(a)(ii)(a)	$\frac{x}{x+3}\times\frac{x-1}{x+2}=\frac{7}{15}$	**B1**	**FT** *their* **(a)(i)** = $\frac{7}{15}$
	$15x(x-1)=7(x+3)(x+2)$	**M1**	Removes all algebraic fractions **FT** *their* equation if in comparable form
	$15x^2-15x=7x^2+21x+14x+42$	**M1**	Correctly expands all brackets **FT** *their* equation if in comparable form
	$[8x^2-50x-42=0]$ $4x^2-25x-21=0$	**A1**	With no errors or omissions seen and one further stage seen after final M1
8(a)(ii)(b)	$(4x+3)(x-7)$ [= 0]	**M2**	**M1** for $4x(x-7)+3(x-7)$ or $x(4x+3)-7(4x+3)$ or for $(4x+a)(x+b)$ where either $ab=-21$ or $4b+a=-25$ If 0 scored, **SC1** for $4x+3$ **and** $x-7$ seen but not in factorised form
	7 and $-\frac{3}{4}$	**B1**	
8(a)(ii)(c)	7	**1**	**FT** *their* positive solution

8(b)	$\frac{1}{6}$ oe	**4**	**M3** for $\frac{5}{9}\times\frac{4}{8}\times\frac{3}{7}+\frac{4}{9}\times\frac{3}{8}\times\frac{2}{7}$ or **M2** for $\frac{5}{9}\times\frac{4}{8}\times\frac{3}{7}$ or $\frac{4}{9}\times\frac{3}{8}\times\frac{2}{7}$ or **M1** for $\frac{5}{9},\frac{4}{8},\frac{3}{7}$ seen or $\frac{4}{9},\frac{3}{8},\frac{2}{7}$ seen If 0 scored, **SC1** for $\frac{5^3+4^3}{729}$ oe

8(a)(i)	$\frac{4}{5}$ oe	**1**	
8(a)(ii)	$\frac{4}{5}$ oe	**1**	
8(b)(i)	$\frac{6}{20}$ oe nfww	**3**	**M2** for $\frac{1}{5}\times\frac{3}{4}+\frac{3}{5}\times\frac{1}{4}$ oe or $2\times\frac{1}{5}\times\frac{3}{4}$ oe or **M1** for $\frac{1}{5}\times\frac{3}{4}$ alone or $\frac{3}{5}\times\frac{1}{4}$ alone or for answer $\frac{3}{20}$ nfww After 0 scored, **SC1** for answer $\frac{6}{25}$
8(b)(ii)	$\frac{8}{20}$ oe nfww	**3**	**M2** for $1-\frac{4}{5}\times\frac{3}{4}$ or $\frac{1}{5}\times1+\frac{4}{5}\times\frac{1}{4}$ oe or $2\times\frac{1}{5}\times1$ or $2\times\frac{1}{5}\times\frac{3}{4}+2\times\frac{1}{5}\times\frac{1}{4}$ or *their* (b)(i) + $2\times\frac{1}{5}\times\frac{1}{4}$ or **M1** for answer $\frac{2\text{ or }4\text{ or }5\text{ or }6\text{ or }7}{20}$ oe nfww After 0 scored, **SC1** for answer $\frac{8}{25}$

8(a)(i)	$\frac{2}{5}$ oe	**2**	**M1** for $\frac{4}{6}\times\frac{3}{5}$
8(a)(ii)	$\frac{3}{5}$ oe	**1**	**FT** 1 – *their* $\frac{12}{30}$ oe
8(b)	$\frac{5}{7}$ oe nfww	**4**	**M3** for $\frac{2}{7}+\frac{5}{7}\times\frac{2}{6}+\frac{5}{7}\times\frac{4}{6}\times\frac{2}{5}$ oe or for $1-\frac{5}{7}\times\frac{4}{6}\times\frac{3}{5}$ oe or **M1** for each of $\frac{5}{7}\times\frac{2}{6}$ and $\frac{5}{7}\times\frac{4}{6}\times\frac{2}{5}$ oe or completed tree diagram with appropriate probabilities shown

9(a)	$\frac{5}{8}$ $\frac{3}{8}$ $\frac{1}{6}$ $\frac{5}{6}$ $\frac{7}{10}$ $\frac{3}{10}$	**3**	**B1** for each pair
9(b)	$\frac{5}{48}$ oe	**2**	**M1FT** for *their* $\frac{5}{8} \times$ *their* $\frac{1}{6}$
9(c)	$\frac{304}{480}$ oe	**3**	**M2** for *their* $\frac{5}{8} \times$ *their* $\frac{5}{6} +$ *their* $\frac{3}{8} \times$ *their* $\frac{3}{10}$ oe or **M1** for *their* $\frac{5}{8} \times$ *their* $\frac{5}{6}$ or *their* $\frac{3}{8} \times$ *their* $\frac{3}{10}$
4(a)(i)	$\frac{8}{20}$ oe	**3**	**M2** for $\frac{2}{5} \times \frac{1}{4} + \frac{3}{5} \times \frac{2}{4}$ or **M1** for one of these products OR **M1** for probability tree identifying all 20 outcomes with the correct 8 identified OR **M1** for completed possibility space / 2-way table identifying the 8 possible outcomes out of 20, oe **SC1** for $\frac{13}{25}$ with replacement
4(a)(ii)	$\frac{9}{25}$ oe	**3**	**M2** for $\frac{2}{5} \times \frac{3}{5} + \frac{3}{5} \times \frac{1}{5}$ oe or **M1** for one of these products OR **M1** for probability tree identifying all 25 outcomes with the correct 9 identified OR **M1** for completed possibility space / 2-way table identifying the 9 possible outcomes out of 25, oe
4(a)(iii)	Jojo and e.g. $\frac{40}{100} > \frac{36}{100}$	**1**	**1FT** *their* **(i)** and **(ii)** dep on being in range 0 to 1
4(b)	$\frac{24}{60}$ oe	**3**	**M2** for $\frac{2}{5} \times \frac{3}{4} \times \frac{1}{3} + \frac{3}{5} \times \frac{2}{4} \times \frac{1}{3} + \frac{3}{5} \times \frac{2}{4} \times \frac{2}{3}$ oe or **M1** for any one correct product OR **M1** for 4, 5, 4 and 5, 4, 4 and 5, 5, 4 clearly identified on a tree or in a list

6(a)(i)	14	**1**	
6(a)(ii)	16	**1**	
6(a)(iii)	$\frac{20}{462}$ oe	**3**	**M2** for $\frac{5}{22}\times\frac{4}{21}$ or **M1** for $\frac{5}{22}$ seen

12(a)(i)	$\frac{10}{20}\times\frac{9}{19}$ oe	**M2**	**B1** for $\frac{9}{19}$ oe seen
12(a)(ii)	$\frac{62}{95}$ oe	**4**	**M3** for $\frac{6}{20}\times\frac{14}{19}+\frac{10}{20}\times\frac{10}{19}+\frac{4}{20}\times\frac{16}{19}$ oe or $1-\frac{6}{20}\times\frac{5}{19}-\frac{10}{20}\times\frac{9}{19}-\frac{4}{20}\times\frac{3}{19}$ oe or **M2** for the sum of two products of different flavours isw or **M1** for one correct product of different flavours isw
12(b)	$\frac{5}{57}$ oe	**3**	**M2** for $N\times\left(\frac{4}{20}\times\frac{3}{19}\times\frac{16}{18}\right)+\frac{4}{20}\times\frac{3}{19}\times\frac{2}{18}$ oe or for $3\left(\frac{4}{20}\times\frac{3}{19}\times\frac{16}{18}\right)$ oe or $1-\{N\times\left(\frac{4}{20}\times\frac{16}{19}\times\frac{15}{18}\right)+\frac{16}{20}\times\frac{15}{19}\times\frac{14}{18}\}$ oe or **M1** for $\frac{4}{20}\times\frac{3}{19}\times\frac{k}{18}$ oe seen

7(a)(i)	$\frac{3}{5} > \frac{1}{4}$ oe or $\frac{12k}{20k}$ and $\frac{5k}{20k}$ or 0.6 and 0.25 or 60% and 25%	**1**	
7(a)(ii)	$\frac{11}{20}$ oe	**3**	**M2** for $\frac{3}{5}\times\frac{3}{4} + \frac{2}{5}\times\frac{1}{4}$ oe or $1 - \frac{3}{5}\times\frac{1}{4} - \frac{2}{5}\times\frac{3}{4}$ oe or **M1** for $\frac{3}{5}\times\frac{3}{4}$ or $\frac{2}{5}\times\frac{1}{4}$ oe (but not as part of a larger product)
7(b)(i)	$\frac{6}{60}$ oe	**2**	**M1** for $\frac{3}{5}\times\frac{2}{4}\times\frac{1}{3}$ oe If 0 scored, **SC1** for answer $\frac{27}{125}$ oe
7(b)(ii)	0	**1**	Accept $\frac{0}{60}$

7(c)	$\frac{11}{25}$ oe	**3**	**M2** for $\frac{3}{5}\times\frac{3}{5} + \frac{2}{5}\times\frac{1}{5}$ oe or $1 - \frac{3}{5}\times\frac{2}{5} - \frac{2}{5}\times\frac{4}{5}$ oe or **M1** for $\frac{3}{5}\times\frac{3}{5}$ or $\frac{2}{5}\times\frac{1}{5}$ or for a correct tree showing all 25 outcomes with the 11 correct outcomes identified

4	**(a)**	$\frac{1}{8}$ oe	**3**	**M2** for $\frac{1}{2}\left(1-\frac{1}{6}-\frac{1}{4}-\frac{1}{3}\right)$ oe or **M1** for $\frac{1}{6}+\frac{1}{4}+\frac{1}{3}$ seen oe or idea that all sum to 1
	(b)	$\frac{7}{12}$ oe	**2**	**M1** for $\frac{1}{3}+\frac{1}{4}$ oe
	(c) (i)	$\frac{1}{16}$ oe	**2**	**M1** for $\frac{1}{4}\times\frac{1}{4}$ oe
	(ii)	$\frac{2}{24}$ oe	**3**	**M2** for $2\times\frac{1}{6}\times\frac{1}{4}$ oe or **M1** for $\frac{1}{6}\times\frac{1}{4}$ oe
	(d)	12	**1**	

6(a)	$\frac{1}{3}, \frac{6}{7}$ correctly placed	**1**	
	$\frac{4}{7}, \frac{3}{7}$ correctly placed	**1**	

6(b)	$\frac{2}{21}$ oe	**2**	**M1** for $\frac{2}{3}\times\frac{1}{7}$
6(c)(i)	$\frac{15}{21}$ oe	**3**	**M2** for $\frac{2}{3}\times\frac{6}{7}+\frac{1}{3}\times\frac{3}{7}$ oe or **M1** for $\frac{2}{3}\times\frac{6}{7}$ oe or $\frac{1}{3}\times\frac{3}{7}$ oe seen
6(c)(ii)	50	**2FT**	**FT** (70 × *their* **(c)(i)**) rounded up or down to integer **M1** for 70 × *their* **(c)(i)**
6(d)	$\frac{10}{243}$ oe	**2**	**M1** for $\frac{2}{3}\times\frac{1}{3}\times\frac{1}{3}\times\frac{1}{3}\times\frac{1}{3}[\times k]$ oe nfww where k is positive integer less than 5

9(a)(i)	52	**2**	**M1** for $(1-0.35)\times 80$ oe
9(a)(ii)	84	**1**	
9(b)(i)	$\frac{27}{729}$ oe	**2**	**M1** for $\frac{3}{9}\times\frac{3}{9}\times\frac{3}{9}$
9(b)(ii)	$\frac{144}{729}$ oe	**3**	**M2** for $\frac{2}{9}\times\frac{3}{9}\times\frac{4}{9}\times 6$ oe or **M1** for $\frac{2}{9}\times\frac{3}{9}\times\frac{4}{9}$ oe isw
9(c)	$\frac{42}{60}$ oe	**4**	**M3** for $\frac{3}{5}\times\frac{2}{4}\times\frac{1}{3}+\frac{3}{5}\times\frac{2}{4}\times\frac{2}{3}\times 3$ oe or **M2** for $\frac{3}{5}\times\frac{2}{4}\times\frac{2}{3}\times 3$ oe or for $\frac{3}{5}\times\frac{2}{4}\times\frac{1}{3}+\left(\frac{3}{5}\times\frac{2}{4}\times\frac{2}{3}\right)[\times 2]$ or **M1** for $\frac{3}{5}\times\frac{2}{4}\times\frac{1}{3}$ or $\frac{3}{5}\times\frac{2}{4}\times\frac{2}{3}$ oe isw or for PPG, PGP, GPP and PPP selected soi

7(a)	$\frac{5}{6}$	**1**	
7(b)	$\frac{4}{36}$ oe	**2**	**M1** for $\frac{2}{6}\times\frac{2}{6}$
7(c)	20	**1**	

7(d)(i)	Diagram completed correctly x x 3 3 3 9 x x 2 2 2 6 x x 2 2 2 6 x x 2 2 2 6 x x 1 1 1 3	**2**	**B1** for 3 correct columns or for 4 correct rows
7(d)(ii)(a)	$\frac{9}{36}$ oe	**1FT**	**FT** *their* **(d)(i)**
7(d)(ii)(b)	$\frac{4}{36}$ oe	**1FT**	**FT** *their* **(d)(i)**
7(e)	$\frac{512}{7776}$ oe	**2**	**M1** for $\left(\frac{4}{6}\right)^k \times \frac{2}{6}$ oe $k = 3$, 4 or 5 only

5 **(a)**	0.05 oe	**2**	**M1** for 1 – (0.2 + 0.3 + 0.45) oe
(b)	15	**1**	
(c) **(i)**	0.75 oe	**2**	**M1** for 0.45 + 0.3 oe
(ii)	0.135 oe	**2**	**M1** for 0.45 × 0.3 oe
(iii)	0.12 oe	**3**	**M2** for 2(0.3 × 0.2) oe or **M1** for 0.3 × 0.2 or 0.06 oe nfww
(d)	0.243 oe	**5**	**M4** for 3(0.45 × 0.45 × 0.2) + 3(0.3 × 0.3 × 0.45) oe or **M3** for 3(0.45 × 0.45 × 0.2) **or** 3(0.3 × 0.3 × 0.45) oe or **M2** for 0.45 × 0.45 × 0.2 **and** 0.3 × 0.3 × 0.45 or **M1** for 0.45 × 0.45 × 0.2 **or** 0.3 × 0.3 × 0.45 oe or for identifying the correct 6 outcomes e.g. 10 0 0, 0 0 10, 0 10 0, 5 5 0, 5 0 5, 0 5 5

4 **(a)** **(i)**	0.0025 or $\frac{1}{400}$ oe	**2**	**M1** for 0.05^2 oe
(ii)	0.9975 or $\frac{399}{400}$ oe	**1FT**	**FT** for 1 – (*their* (a)(i)) oe
(b)	0.171 or 0.1714 to 0.1715 or $\frac{6859}{40\,000}$	**3**	**M2** for $4(0.05 \times 0.95^3)$ oe **M1** for 0.05×0.95^3 oe seen or for the 4 combinations correctly identified

	(c)	376 nfww	**4**	**M1** for midpoints soi (condone 1 error or omission) (225, 275, 325, 375, 425, 475) and **M1** for use of Σfx with x in correct interval including both boundaries (condone 1 further error or omission) and **M1** (dependent on second M) for $\Sigma fx \div 200$
	(d) (i)	16	**1**	
	(ii)	33	**2**	**M1** for $0.8 \times 50 + 0.26 \times 100$

7	**(a)**	0.7, 0.1 oe correctly placed 0.2, 0.8 oe correctly placed	**1** **1**	
	(b) (i)	0.44 nfww oe	**3**	**M2** for $1 - \textit{their}\ 0.7 \times \textit{their}\ 0.8$ or for $0.3 + \textit{their}\ 0.7 \times \textit{their}\ 0.2$ oe or **M1** for $\textit{their}\ 0.7 \times \textit{their}\ 0.8$ or for two of 0.3×0.9, $0.3 \times \textit{their}\ 0.1$, $\textit{their}\ 0.7 \times \textit{their}\ 0.2$
	(ii)	110	**1FT**	**FT** $250 \times \textit{their}$ **(b)(i)**
	(c)	If late at first two stations then certain to be late at station C oe	**1**	Indication of certain event (allow 1 or 100% probability or sure) at third station if late at first two stations

5	(a) (i)	$\frac{3}{4}$, $\frac{1}{4}$ $\frac{7}{8}$, $\frac{1}{8}$	2	**B1** for any 2 correct
	(ii)	$\frac{21}{32}$ oe	2	**M1** for $\frac{7}{8}\times\frac{3}{4}$ oe
	(iii)	$\frac{441}{1024}$ oe	2FT	**M1** for $\left(\frac{7}{8}\times\frac{3}{4}\right)^2$ or *their* **((a)(ii))**^{2} oe
	(b)	175	2	**M1** for $200 \times \frac{7}{8}$
	(c)	2400	2	**M1** for 1575 ÷ *their***(a)(ii)**

5	(a)	$\frac{1}{64}$	2	**M1** for $\frac{1}{8} \times \frac{1}{8}$
	(b)	$\frac{63}{64}$	1FT	**FT** 1 – *their* **(a)**
	(c)	$\frac{30}{64}$ oe	2	**M1** for [2 ×] $\frac{3}{8} \times \frac{5}{8}$ oe
	(d)	$\frac{7}{64}$	3	**M2** for $\frac{1}{8} \times \frac{1}{8} + \frac{1}{8} \times \frac{3}{8} + \frac{3}{8} \times \frac{1}{8}$ oe or **M1** for identifying combinations required, (8, 8) and (8, 6) and (8, 5) or identifying 6 out of the 7 possible outcomes
	(e)	$\frac{24}{64}$ oe	3	**M2** for $\frac{1}{8} \times \frac{7}{8} + \frac{3}{8} \times \frac{4}{8} + \frac{2}{8} \times \frac{2}{8} + \frac{1}{8} \times \frac{1}{8}$ oe or $\frac{7}{8} \times \frac{1}{8} + \frac{6}{8} \times \frac{1}{8} + \frac{4}{8} \times \frac{2}{8} + \frac{1}{8} \times \frac{3}{8}$ oe or **M1** for the sum of any two correct products from above oe isw